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Duality, Equivalence, Mass and The Quest For The Vacuum [a]

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I contemplate the possibility that the mismatch between the maximally symmetric point (the free fermionic point) and the strictly self–dual point in the Narain moduli space plays a role in the string vacuum selection. The role of self–duality in the recent formulation of quantum mechanics from an equivalence postulate, and the new perspective that it offers on the foundations of quantum gravity and the origin of mass, are discussed.

The central issue in elementary particle physics is the nature of the electroweak symmetry breaking mechanism. Perhaps not unrelated, and of equal importance, is the formulation of the consistent synthesis of gravity and quantum mechanics from fundamental physical postulates. String theory and its non–perturbative generalizations constitute the most advanced such attempts.

The Standard Model, and many of its contemporary theoretical extensions, utilize fundamental scalar representations to break the electroweak symmetry. To my knowledge all of the existing string theories give rise to fundamental scalar representations. Therefore, as a matter of classification we may classify all the string theories as theories that include fundamental scalars. The precise realization of the scalar state in nature, and its role in electroweak symmetry breaking is, however, at present an experimentally unresolved issue.

The experimental success of the Standard Model raises the problem of understanding the origin of its structure and parameters. The Standard Model multiplets suggest the embedding in Grand Unified Theories. The flavor sector does not have an appealing explanation in this framework, and necessitates further ad–hoc assumptions. A more constraining framework is sought in the

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context of theories which unify gravity with the gauge interactions. Such a concrete framework is given by string theories. In this conference the talks by Fernando Quevedo and Jerry Cleaver report on some current efforts. Since my work in this area is also covered in Cleaver's talk, I only discuss here briefly some of the interesting aspects.

String theories give rise to a huge number of potentially viable vacua. Selecting the correct one among them is a daunting task. One may further question whether in the lack of complete understanding of the theory such an endeavor is not futile to begin with. However, with present day understandings a reasonable goal is to use the low energy data to single out the string theories that most closely resemble the real world. The important guide in this quest is the multiplet structure of the Standard Model. It is natural to seek superstring models which preserve the $SO(10)$ embedding of the Standard Model spectrum. The $SO(10)$ symmetry however can be broken directly at the string theory level rather than at the level of the effective field theory. Additionally we must impose the existence of three chiral generations.

The heterotic–string models constructed in the free fermionic formulation naturally achieve both of these criteria[1]. Furthermore, a generic consequence of perturbative string models is the existence of numerous massless states beyond the spectrum of the Minimal Supersymmetric Standard Model. Many of these states carry fractional electric charge and consequently must be decoupled from the low energy spectrum. Recently, it was demonstrated, in the FNY heterotic string model[2], that free fermionic models also give rise to models in which all the states beyond the MSSM decouple from the low energy spectrum at or slightly below the string scale[3]. More on this is discussed in Cleaver's talk.

It should be emphasized that the success of the FNY model in producing a Minimal Standard Heterotic String Model should not be viewed as implying that the FNY model is the correct string vacuum. Indeed, it is preposterous at present to suggest that any three generation string model is the true string vacuum. The free fermionic models, however, give rise to a large class of three generation models. Therefore, it makes sense to extract the features that underly this large class of models. While it is preposterous to suggest that any string model is the true string vacuum, it is not unplausible that the true string vacuum shares some of the properties of the realistic free fermionic models. It is these properties that we would like to extract. It is also quite plausible that these underlying properties may also offer a clue to the dynamical mechanism which selects the string vacuum. It is important to remark that the eventual true string vacuum need not necessarily be an heterotic string. Indeed, it may not even be a string at all! However, the heterotic string may still provide a useful probe to the vital properties of the true vacuum.

The free fermionic models are built by specifying a set of boundary condition basis vectors for all world–sheet fermions and the one–loop GSO projection coefficients [4]. The NAHE set, $\{\mathbf{1}, S, b_1, b_2, b_3\}$, is a set of boundary condition basis vectors which is common in all the realistic free fermionic models. The important aspect of the NAHE set is its correspondence with $Z_2 \times Z_2$ orbifold compactification [5]. However, this $Z_2 \times Z_2$ orbifold act at a very special point in the Narain moduli space! At this point the symmetry which arises from the six dimensional compactified lattice is maximized! That is, at the point in the Narain moduli space where the internal compactified dimensions can be represented as free fermions propagating on the string world–sheet one obtains the maximal symmetry, which for T^6 is $SO(12)$.

The free fermionic point in the Narain moduli space is a maximally symmetric point. However, as is well known toroidal compactifications of string theories possess a duality symmetry under the interchange of winding and momentum modes and the generalization of $R \leftrightarrow 1/R$ duality [6]. The free fermionic point is realized for a specific value of R. Thus, the maximally symmetric point is realized at a specific value of R in the moduli space. Under the duality interchange there exist a value for R which is self–dual. The strictly self–dual point is realized at the point with $(G+B)^2 = I$, where G and B are the metric and antisymmetric tensor, respectively. The enhanced symmetry at the strictly self–dual point of a six dimensional compactified torus is $SU(2)^6$. We see that there is a mismatch between the most symmetric point, where the internal dimensions are realized as free fermions on the world–sheet, and the strictly self–dual point.

We can contemplate that the mismatch between the maximally symmetric point and the strictly self–dual point plays a role in the dynamical mechanism which selects the string vacuum. One possibility (which I am not sure is correct, but can be examined explicitly by studying the relevant partition functions) is that the effect of the orbifold twisting is to move the free fermionic point to the self–dual point. It is interesting to note that after the $Z_2 \times Z_2$ orbifold twisting the $SO(12)$ symmetry is broken to $SU(2)^6$, which is the enhanced symmetry at the strictly self–dual point. This scenario would then suggest a dynamical reason why the $Z_2 \times Z_2$ orbifold is selected. There are however several caveats to this proposal. The first is that supersymmetry is unbroken. We may envision the possibility that supersymmetry is broken dynamically by hidden sector condensation rather than by the mechanism which selects the string vacuum. In this regard it would also be of interest to study the properties of self–dual string models that are not supersymmetric. A second caveat is that the argument above is in the framework of perturbative string compactifications. However, from string dualities we know that there is a non-perturbative structure which underlies the different string theories. Moreover, we know that in this structure an eleventh dimension plays a key role. We may envision the possibility that the argument holds also for the eleventh dimension. This would seem to suggest why the Horava–Witten theory [7] is the viable framework. However, it would also suggest that string coupling is of order one, which seems to be in contradiction with the coupling extracted from extrapolation of the gauge couplings from low energies which yield a smaller value. The possible resolution may be the existence of additional vector–like matter states, beyond the MSSM, in the desert [8].

Duality and self–duality also play a key role in the recent formulation of quantum mechanics from an equivalence postulate [9,10,11]. This is seemingly unrelated to the string program. However, I suggest that this is not the case. As expounded above the central issues of particle physics are the problem of mass and the formulation of quantum gravity from fundamental postulates. Although string theory provides a useful probe for quantum gravity, surely it does not yet provide such a satisfactory formulation, even with the deeper understandings gained from string dualities. Moreover, at the basic observational level none of the current approaches to quantum gravity provides a compelling solution to the vacuum energy problem. It seems to me that all of the current approaches entail, in one form or another, a careful bookkeeping of the energy checkbook, and therefore in the end amount to some form of fine tuning. However, rather than a slight adjustment what may be needed is a new Copernican revolution, which in our case would be a new view of the Hilbert space. I propose that the formulation of quantum gravity from the equivalence postulate offers such a new view. A key question in this respect is, how do the basic particle properties arise in this formalism.

An important facet of the equivalence postulate derivation is the phase–space duality, which is manifested in this formalism due to the involutive nature of the Legendre transformation. The phase–space duality arises due to the defining relation between the dual variables, $p = \partial_q \mathcal{S}_0$, through the generating function $\mathcal{S}_0$. However, the fact that the Legendre transformation is not defined for linear functions, *i.e.* for physical systems with $\mathcal{S}_0 = Aq + B$, implies that the Legendre duality fails for the free system and for the free system with vanishing energy. The Legendre phase–space duality and its breakdown for the free system are intimately related to the equivalence postulate, which states that all physical systems labeled by the function $\mathcal{W}(q) = V(q) - E$, can be connected by a coordinate transformation, $q^a \to q^b = q^b(q^a)$, defined by $\mathcal{S}_0^b(q^b) = \mathcal{S}_0^a(q^a)$. This postulate implies that there always exist a coordinate transformation connecting any state to the state $\mathcal{W}^0(q^0) = 0$. Inversely, this means that any physical state can be reached from the state $\mathcal{W}^0(q^0)$ by a coordinate transformation. This postulate cannot be consistent with classical mechanics. The reason being that in Classical Mechanics (CM) the state $\mathcal{W}^0(q^0) \equiv 0$ remains a fixed point under coordinate transformations. Thus, in CM it is not possible to generate all states by a coordinate transformation from the trivial state. Consistency of the equivalence postulate implies the modification of CM, which is analyzed by a adding a still unknown function Q to the Classical Hamilton–Jacobi Equation (CHJE). Consistency of the equivalence postulate fixes the transformation properties for $\mathcal{W}(q)$,

$$\mathcal{W}^v(q^v) = (\partial_{q^v} q^a)^2 \, \mathcal{W}^a(q^a) + (q^a; q^v),$$

and for $Q(q)$,

$$Q^v(q^v) = (\partial_{q^v} q^a)^2 \, Q^a(q^a) - (q^a; q^v),$$

which fixes the cocycle condition for the inhomogeneous term

$$(q^a; q^c) = \left(\partial_{q^c} q^b\right)^2 [(q^a; q^b) - (q^c; q^b)].$$

The cocycle condition is invariant under Möbius transformations and fixes the functional form of the inhomogeneous term. The cocycle condition is generalizable to higher, Euclidean or Minkowski, dimensions, where the Jacobian of the coordinate transformation extends to the ratio of momenta in the transformed and original systems.

The identity

$$(\partial_q \mathcal{S}_0)^2 = \hbar^2/2 \left(\{\exp(i2\mathcal{S}_0/\hbar, q)\} - \{\mathcal{S}_0, q\}\right)$$

is the second key ingredient in the equivalence postulate formulation. Making the identification

$$\mathcal{W}(q) = V(q) - E = -\hbar^2/4m\{\mathrm{e}^{(i2\mathcal{S}_0/\hbar)}, q\},$$

and

$$Q(q) = \hbar^2/4m\{\mathcal{S}_0, q\},$$

we have that $\mathcal{S}_0$ is solution of the Quantum Stationary Hamilton–Jacobi Equation (QSHJE),

$$(1/2m)\left(\partial_q S_0\right)^2 + V(q) - E + (\hbar^2/4m)\{\mathcal{S}_0, q\} = 0,$$

where $\{,\}$ denotes the Schwarzian derivative. From the identity we deduce that the trivializing map is given by $q \to \tilde{q} = \psi^D/\psi$, where ψ^D and ψ are the two linearly independent solutions of the corresponding Schrödinger equation[9].

We see that the consistency of the equivalence postulate forces the appearance of $\hbar$ as a covariantizing parameter.

The remarkable property of the QSHJE, which distinguishes it from the classical case, is that it admits non–trivial solution also for the trivial state, $\mathcal{W}(q) \equiv 0$. In fact the QSHJE implies that $\mathcal{S}_0 = constant$ is not an allowed solution. The fundamental characteristic of quantum mechanics in this approach is that $\mathcal{S}_0 \neq Aq + B$. Rather, the solution for the ground state, with $V(q) = 0$ and $E = 0$, is given by

$$\mathcal{S}_0 = i\hbar/2 \ln q,$$

up to Möbius transformations. Remarkably, this quantum ground state solution coincides with the self–dual state of the Legendre phase–space transformation and its dual. Thus, we have that the quantum self–dual state plays a pivotal role in ensuring both the consistency of the equivalence postulate and definability of the Legendre phase–space duality for all physical states. The association of the self–dual state and the physical state with $V(q) = 0$ and $E = 0$ provides a hint that the equivalence postulate and Legendre phase–space duality may shed new light on the nature of the vacuum.

A second remarkable consequence of the equivalence postulate is that it implies energy quantization for bound states without assuming the probability interpretation of the wave–function. Consistency of the equivalence postulate implies that the trivializing map, $q \to \tilde{q} = \psi^D/\psi$ should be continuous on the extended real line. It is then seen that this condition is synonymous to the requirement that the physical solution of the corresponding Schrödinger equation admits a square integrable solution, without assuming the probability interpretation of the wave function. The equivalence postulate formalism may therefore indeed offer an entire new perspective on the origin of the Hilbert space structure. The relation of the formalism to unifirmization theory and Riemann surfaces suggests that the Hilbert space structure has a quantum–gravitational origin.

The equivalence postulate derivation may also shed light on the quantum origin of mass. The generalization of the Schwarzian identity to the relativistic case with a vector potential is,

$$\alpha^2(\partial\mathcal{S} - eA)^2 = D^2(Re^{\alpha\mathcal{S}})/(Re^{\alpha\mathcal{S}}) - \Box R/R - (\alpha/R^2)\partial \cdot (R^2(\partial\mathcal{S} - eA)),$$

where $\alpha = i/\hbar$, D is a covariant derivative, and $\partial \cdot (R^2(\partial\mathcal{S} - eA)) = 0$ is a continuity condition. The $D^2(Re^{\alpha\mathcal{S}})/(Re^{\alpha\mathcal{S}})$ term is associated with the Klein–Gordon equation. In this case $\mathcal{W}(q) = 1/2mc^2$. From the equivalence postulate it follows that masses of elementary particles arise from the inhomogeneous term in the transformation of the $\mathcal{W}^0(q^0) \equiv 0$ state, *i.e.*

$$1/2mc^2 = (q^0; q).$$

From this perspective we may speculate that scalar particles and symmetry breaking represent a particular realization of the geometrical transformation $q^0 \to q$. Obviously, this interpretation offers new possibilities to understand how particle properties are generated from the vacuum. Generalizing the Schwarzian identity to curved space will provide the equivalence postulate approach to quantum gravity. Similarly, the identity can be extended to include fermions. The more interesting question, however, is to understand how the fermionic degree of freedom, which has no classical counterpart, arises from the consistency of the equivalence postulate. We anticipate, once again, that the clue is given in the identity itself.

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